GENERAL RELATIVITY AND SPATIAL FLOWS: II. THE HOLLOW SHELL CAVENDISH EXPERIMENT*

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Abstract

The internal gravitational fields of bodies which are predicted by General Relativity and the spatial flow theory of gravity are compared. In contrast to the case of the external fields, the internal fields in the two theories are completely different for ordinary states of matter. We discuss the details of these startling differences and suggest a simple, and yet pivotal, hollow shell Cavendish experiment which can easily discern between them. The parallels between General Relativity and the spatial flow theory are made for the case of extraordinary states of matter.

1. Introduction

In previous articles [1,2,3], we have been investigating the idea that gravitation might be related to the flow and expansion of space (acting as an underlying physical substratum). Instead of the more familiar General Relativistic picture of space being essentially static and curved around gravitational attractors, we have been studying the alternative and General Relativistically viable explanation that space is flowing into or out of ordinary planetary bodies, that it is swirling and spiraling in much larger galactic type structures, and that out in the great expanses of the Universe, its speed might be superluminal in various circumstances. If it turns out that Nature sometimes prefers flowing space to curved space, it would certainly change the way we think about the foundations of physics and raise questions about our basic assumptions in cosmology.

The idea that space might be flowing into or out of the planets is not entirely new. In the 1930's and 1940's, Herbert Ives [4] theoretically investigated clocks and

* http://www.gravityresearch.org/pdf/GRI-010515.pdf

interferometers immersed in an Earthly aetherial inflow and found their behavior to be the same as that predicted by the static and curved (exterior) Schwarzschild solution. In the 1950's, Robert Kirkwood [5] investigated a similarly improvised inflow model in which he used Huygens' Principle and hydrodynamic kinematics to derive several of the standard General Relativistic effects.

In comparison, our papers [1,2,3] have proven that the spatial inflow and outflows are *entirely* physically equivalent to the exterior Schwarzschild solution in the case of an isolated gravitational attractor. This has been accomplished by using the Principle of Covariance within the context of General Relativity itself.[1] We have also attempted to build a physically intuitive bridge between the Einsteinian and Lorentzian (absolute space) views of relativity which works not only for Special Relativity but for the case of General Relativity and gravitation as well. Most importantly, we have suggested two new types of satellite experiments which are capable of discerning the difference between curved space and flowing space. We briefly review them here.

In the case of the isolated attractor, we know from the Principle of Covariance that there is no physical experiment which can be used to tell the difference between the curved and static Schwarzschild representation and its equivalent Galilean coordinate inflow and outflow representations. They are all physically equivalent. (It surprises most people that the *outflow* representation attracts matter and bends light in the same way as the inflow representation.)

In the two-body case (say, the Earth-Sun system), we wonder if Nature prefers the flowing type solutions to the static and curved type solutions. This really comes down to asking if there are different boundary conditions that have to be satisfied in the two-body case (which, in turn, is equivalent to requiring certain algebraic coordinate conditions on the space-time metric). When we bring the flowing solutions of the two bodies together, we get a *gravitopause* (a hyperboloid-like surface of revolution) which separates the flows of the two bodies (this is represented by the dashed line in figure (1-1)).[2] In the case of the idealized Earth-Sun system rotating around its barycenter, there will be a region of stagnation between the two flows somewhat near the saddle point of the

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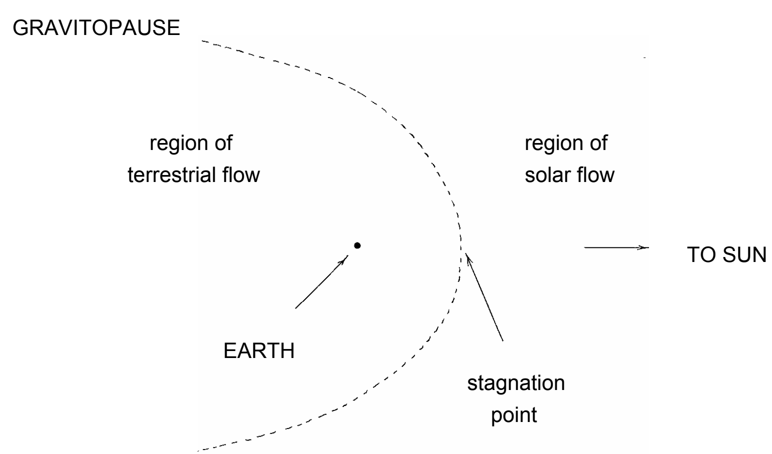
[1] We have since learned that the space-time metric which we have used for the flow representations of the Schwarzschild solution was originally discovered by Painlevé and Gullstrand in the early 1920's. However, these authors seem to have not given it the same physical significance.

[2] The reader may wish to compare this figure with the one in Clerk Maxwell's letter to Faraday [6].

superposed Newtonian gravitational potentials (it is possible that the stagnation region will be turned somewhat in the direction of Earth's orbit).[3] This region is on the order of 260,000 km from the Earth (approximately 40 Earth radii). There will be no time dilation experienced by a clock which is situated at rest at the stagnation point of the flow solution. This differs from the predicted time dilation in the usual curved space two-body solution by about a part in 10^8 (easily measurable). There will also be time dilation jumps on the order of a part in 10^8 whenever a satellite passes through the gravitopause anywhere in the region shown in figure (1-1). This is because the flow is about 1 km/sec just on the terrestrial side and about 42 km/sec just on the solar side. The onboard clock has to be freely running (unlocked from the telemetry transponder).



(1-1)

These *time dilation* experiments were the first of the new satellite experiments we suggested. They might be the first experiments in which the *magnitude* of the translational flow of space is measured. The second type of satellite experiments we suggested had to do with the geodesic *trajectories* of the satellites (as predicted by General Relativity) as the satellites pass through the gravitopause. For example, there is a possible Coriolis-type deflection in the direction of the solar flow and a slight forward accelerative bump on the satellite as it crosses from the terrestrial side to the solar side. Experiments of this second type might be the first to measure the actual *direction* of the translational flow of space.

There have already been satellites flown in which these time dilation and orbital experiments could have easily been incorporated. Refer, for example, to the perfectly

[3] This saddle point is not to be confused with the L1 Lagrange point which includes the centrifugal potential and which is located approximately 1.5 million km from the Earth in the direction of the Sun.

placed petal orbit [7] flown by NASA's solar wind experiment WIND from November 1998 to April 1999.

However, before we undertake the effort which would be involved in these satellite experiments, it turns out that we can determine whether or not spatial flow is physically real and essential to ordinary gravitational phenomena by carrying out a simple, and yet pivotal, laboratory experiment. It is the intent of this present paper to demonstrate how this can be achieved by means of a Cavendish-type apparatus which explores the *interior* of a hollow homogeneous spherical shell in the radial direction. If the reality of spatial flow is confirmed in this simple laboratory experiment, we will have added impetus for bringing the aforementioned satellite experiments to fruition.

In the case of the *exterior* Schwarzschild gravitational solution for ordinary gravitational attractors, our previous work has demonstrated that spatial flow and General Relativity are essentially physically equivalent. We will discover in what follows that the situation for the *interior* solutions is just the opposite. For the type of stress tensors encountered in ordinary states of matter, spatial flow and General Relativity predict entirely different behaviors for test bodies in the interior regions of gravitational attractors. Thus, the proposed hollow shell Cavendish experiment will be a watershed experiment in which either spatial flow is affirmed and General Relativity is refuted, General Relativity is affirmed and spatial flow is refuted, or both General Relativity and spatial flow are refuted.

2. Spatial Flows as Interior Solutions of Einstein's Equations

In the spatial flow approach to space-time physics, every space-time manifold of physical significance is supposed to be capable of being characterized by a flow of physical space (refer to the references in footnote 1). A global Galilean coordinate frame $\{\mathbf{r},t\}$ exists in which the flow is represented by a global 3-space vector field $\mathbf{w} \equiv \mathbf{w}(\mathbf{r},t)$. The rate of an atomic clock depends on its *absolute speed* u with respect to this physical space. Thus, in these Galilean coordinates, the proper time element of an atomic clock is given by

$$d\tau = \gamma^{-1}\, dt \equiv \sqrt{1-u^2/c^2}\; dt = \sqrt{1-\mathbf{u}\cdot\mathbf{u}/c^2}\, dt \quad , \tag{2-1}$$

where

$$\mathbf{u} \equiv \mathbf{v} - \mathbf{w} \tag{2-2}$$

is the *absolute velocity* of the clock relative to physical space. Here, τ is the proper time of the clock, c is the speed of light with respect to physical space (a constant), t is the coordinate time, $\mathbf{r}$ is the coordinate position vector of the clock, and $\mathbf{v} \equiv d\mathbf{r}/dt$ is the coordinate velocity of the clock.

From the above equations, we see that the space-time line element in Galilean coordinates with arbitrary spatial flow $\mathbf{w}$ always takes the form

$$c^2 d\tau^2 = (c^2 - w^2)dt^2 + 2\mathbf{w} \cdot d\mathbf{r}\, dt - (d\mathbf{r})^2 . \tag{2-3}$$

In four dimensional *rectangular* Galilean coordinates $\{x^0, x^1, x^2, x^3\} \equiv \{ct, x, y, z\}$, the components of the corresponding space-time *metric tensor* are seen to be

$$g_{\kappa\lambda} \quad = \quad \begin{bmatrix} -(1 - w^2/c^2) & -w^x/c & -w^y/c & -w^z/c \\ -w^x/c & 1 & 0 & 0 \\ -w^y/c & 0 & 1 & 0 \\ -w^z/c & 0 & 0 & 1 \end{bmatrix} \quad , \tag{2-4}$$

where we use the convention that

$$ds^2 \equiv c^2 d\tau^2 \equiv -g_{\kappa\lambda}\, dx^\kappa\, dx^\lambda \quad . \tag{2-5}$$

From (2-3) or (2-4), it is immediately evident that every slice of space-time with constant time is flat 3-dimensional Euclidean space. We also realize that the spatial flow approach to General Relativity corresponds to the possibility of always being able to impose algebraic *coordinate conditions* in the guise of the special form of the metric tensor in (2-3) and (2-4).

In this paper, we will only be interested in spherically symmetric spatial flows $\mathbf{w}$

and spherically symmetric solutions to Einstein's field equations

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = - \frac{8\pi G}{c^4} T_{\mu\nu} \quad , \tag{2-6}$$

where $G_{\mu\nu}$ is the Einstein tensor, G is the gravitational constant, and $T_{\mu\nu}$ is the energy-momentum-stress tensor. Thus, we shall use a *spherical* Galilean coordinate frame $\{x^0, x^1, x^2, x^3\} \equiv \{t, r, \theta, \phi\}$ in which the steady state spherically symmetric flow metric takes the form

$$g_{\mu\nu} = \begin{bmatrix} -(1-w^2) & -w & 0 & 0 \\ -w & 1 & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{bmatrix} \quad , \tag{2-7}$$

where $w = w(r)$ is the time-independent radial component of the spatial flow, and where we have simplified things by choosing space and time measurement units so that $c \equiv 1$. One can use a symbolic manipulation program [8] to conveniently calculate the Einstein tensor which results as a consequence of the constraint (2-7) on the form of the space-time metric. The result is

$$G_{\mu\nu} = \begin{bmatrix} -(1-w^2)\frac{1}{r^2}\frac{d(rw^2)}{dr} & -w\frac{1}{r^2}\frac{d(rw^2)}{dr} & 0 & 0 \\ -w\frac{1}{r^2}\frac{d(rw^2)}{dr} & \frac{1}{r^2}\frac{d(rw^2)}{dr} & 0 & 0 \\ 0 & 0 & r^2\frac{1}{2}\nabla^2(w^2) & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta\frac{1}{2}\nabla^2(w^2) \end{bmatrix} \tag{2-8}$$

where ∇^2 denotes the usual Laplacian operator $\mathrm{lap} \equiv \mathrm{div}\,\mathbf{grad}$. The Einstein tensor corresponding to a spherically symmetric flow *always* has this form in spherical Galilean coordinates.

Since

$$g^{\mu\nu} = \begin{bmatrix} -1 & -w & 0 & 0 \\ -w & (1-w^2) & 0 & 0 \\ 0 & 0 & r^{-2} & 0 \\ 0 & 0 & 0 & r^{-2}\csc^2\theta \end{bmatrix} \quad , \tag{2-9}$$

we have as a consequence of equations (2-6), (2-8), and (2-9) that the radial stress is necessarily everywhere equal to the mass-energy density:

$$T_1^{\ 1} = T_0^{\ 0} \quad . \tag{2-10}$$

Thus, it is a consequence of General Relativity that the algebraic coordinate conditions imposed on the space-time metric by the requirement that it have the form corresponding to a spherically symmetric steady state spatial flow always imply that the radial stress is equal to the mass-energy density for any spherically symmetric mass distribution. When the radial stress and mass-energy density are not zero (in other words, *inside* the mass distribution) this represents a very extraordinary and highly improbable state of matter. In contrast, *outside* the spherically symmetric mass distribution, there is nothing extraordinary. Wherever we have the usual *exterior* radial flow $w(r) = \sqrt{2GM/r}$ representing the exterior Schwarzschild solution, for example, we find that the Einstein tensor and the energy-momentum-stress tensor are always zero, as expected.

To end this Section, we emphasize what we have discovered. For ordinary states of matter such as one might expect to find in a normal spherically symmetric planet or in a normal laboratory object, the interior gravitational fields cannot correspond to General Relativity and at the same time be characterized by a spatial flow.

3. The Action-at-a-distance Gravitational Physics of a Hollow Spherical Shell

Since General Relativity and spatial flow are incompatible in the interior regions of ordinary states of matter, we are naturally curious about the differences in the structure of the gravitational fields they predict for these interior regions. We shall see that the simple hollow spherical shell is very effective for highlighting these differences.

The Newtonian action-at-a-distance gravitational behavior of a homogeneous spherical shell of constant mass density μ has been known for centuries. For comparison purposes, we shall review the traditional derivation which is standard in potential theory. One starts with Poisson's equation,

$$\operatorname{lap}\psi \equiv \nabla^2\psi = 4\pi G\mu \quad , \tag{3-1}$$

as the *field equation* for the Newtonian gravitational potential ψ associated with an arbitrary mass distribution $\mu = \mu(\mathbf{r})$. The gravitational field (the gravitational force per unit of test mass) associated with this mass distribution is then computed as the gradient of the potential,

$$\mathbf{g} = -\,\mathbf{grad}\,\psi \equiv -\nabla\psi \quad . \tag{3-2}$$

The general solution of the inhomogeneous partial differential equation (3-1) in any particular region of space is the sum

$$\psi = \psi_P + \psi_H \tag{3-3}$$

of the particular integral solution

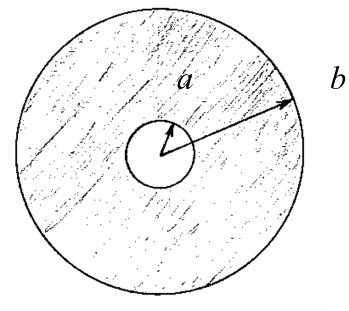
$$\psi_P(\mathbf{r}) \equiv -G \iiint_{V'} \frac{\mu(\mathbf{r}')}{R}\, dV' \quad , \tag{3-4}$$

of Poisson's equation (the integration region V' is all of space and $R \equiv |\,\mathbf{r} - \mathbf{r}'\,|$) and any solution ψ_H of the corresponding homogeneous Laplace equation,

$$\operatorname{lap}\psi_H \equiv \nabla^2\psi_H = 0 \quad . \tag{3-5}$$

The actual *physical* solution ψ one ends up with depends on the choice of the homogeneous solution ψ_H which is added to the particular integral solution ψ_P. What one chooses for ψ_H is determined by the physical *boundary conditions* which are assumed to be correct for the potential ψ.

For a homogeneous shell of inner radius a and outer radius b of constant mass density μ,



(3-6)

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the traditional derivation [9] makes the assumption that

$$\psi_H \equiv 0 \quad , \tag{3-7}$$

so that the physical solution is just the integral solution (3-4). The result is as follows.

<u>outside the shell</u> $(r > b)$:

$$\psi_P(r > b) = -\frac{GM}{r} \equiv -2\pi G\mu \frac{2(b^3 - a^3)}{3r} \tag{3-8a}$$

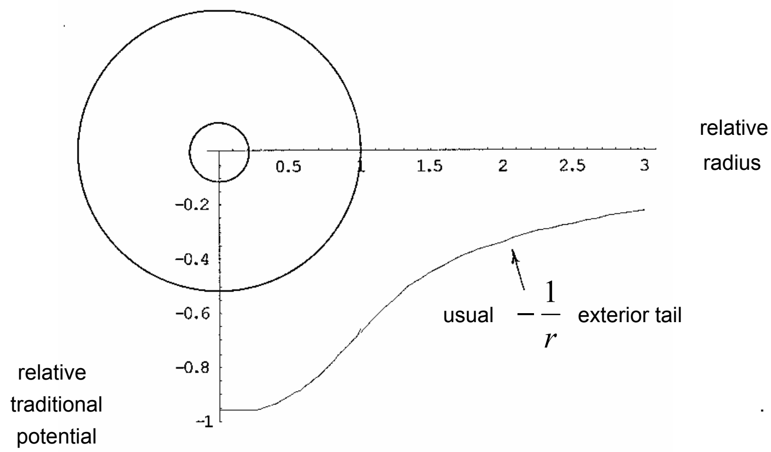
<u>within the shell</u> $(a \le r \le b)$:

$$\psi_P(a \le r \le b) = -2\pi G\mu\left(b^2 - \frac{2a^3}{3r} - \frac{r^2}{3}\right) \tag{3-8b}$$

<u>inside the cavity of the shell</u> $(r < a)$:

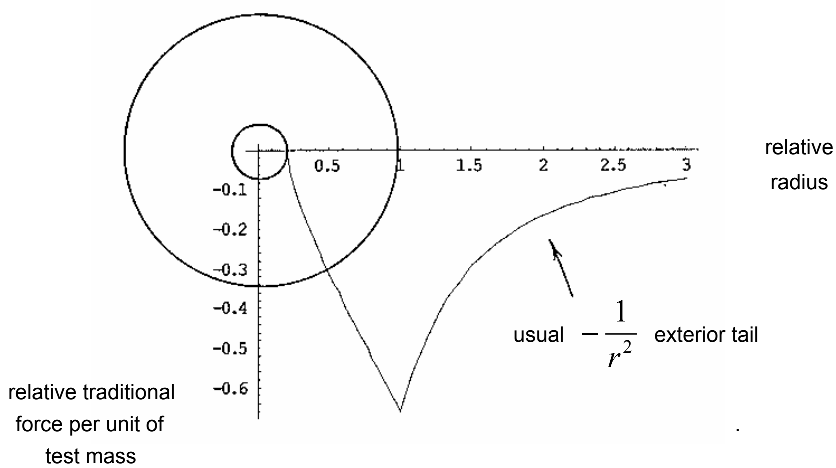
$$\psi_P(r < a) = -2\pi G\mu(b^2 - a^2) \qquad . \tag{3-8c}$$

We can get a qualitative view of this traditional potential by dropping the $2\pi G\mu$ factor and plotting the relative potential normalized so that the outer radius of the shell is $b \equiv 1$ and the cavity radius is, for example, $a \equiv 0.2$:



(3-9)

The corresponding traditional gravitational attraction of the shell is obtained by taking the derivative of this traditional potential. A plot of it normalized in a similar fashion looks like this:



(3-10)

The fact that it is negative all the way down to the cavity radius $a = 0.2$ means that it is an *attractive* force all the way down to the cavity. Inside the cavity, the gravitational force is always zero. Outside the shell, it acts as if it were the inverse square attraction of the total mass of the shell M acting as a point source at the origin. All of this is mother's milk to physicists. It is the gravitational behavior of a homogeneous spherical shell according to Newton's *action-at-a-distance* inverse square law of attraction. With the choice of the homogeneous function (3-7), the potential theory derivation corresponds exactly to the inverse square action-at-a-distance formulation,

$$\mathbf{g}(\mathbf{r}) \; = \; -G \iiint_{V'} \frac{\mu(\mathbf{r}')}{R^2} \, \hat{\mathbf{e}}_R \; dV' \quad , \tag{3-11}$$

where $\mathbf{R} \equiv \mathbf{r} - \mathbf{r}'$ and $\hat{\mathbf{e}}_R \equiv \mathbf{R} / |\mathbf{R}|$.

4. The Spatial Flow Gravitational Physics of a Hollow Spherical Shell

Newton, himself, recognized that action-at-a-distance gravitational physics made no physical sense. In his oft quoted letter to Bentley, he wrote [10]

> "It is inconceivable, that inanimate brute matter should, without the mediation of something else, which is not material, operate upon, and affect other matter without mutual contact; as it must do, if gravitation, in the sense of Epicurus, be essential and inherent in it. And this is one reason, why I desired you would not ascribe innate gravity to me. That gravity should be innate, inherent, and essential to matter, so that one body may act upon another, at a distance through a vacuum, without the mediation of anything else, by and through which their action and force may be conveyed from one to another, is to me so great an absurdity, that I believe no man who has in philosophical matters a competent faculty of thinking, can ever fall into it. Gravity must be caused by an agent acting constantly according to certain laws; but whether this agent be material or immaterial, I have left to the consideration of my readers."

We agree with Newton. Physicists must look for a local and underlying agent which is the cause of gravity. Every avenue must be explored. One significant possibility is that gravity is caused by the immaterial flow and expansion of space. This has been the essential theme of our recent investigations [1,2,3]. As we proceed with the development of this spatial flow hypothesis, we are going to discover that it leads to some rather startling predictions.

What can we expect for the flow of space associated with a typical homogeneous hollow spherical shell? For $r > b$, or in other words, beyond the outer surface of the shell (3-6), we know that the square of the speed of the spatial flow falls off as $1/r$ out to infinity (see, particularly, reference [3]). This is the usual exterior spatial flow solution. We know that the temporal gravitational acceleration of a test mass depends on the spatial inhomogeneity of the spatial flow. Beyond the outer surface of this mass shell, the speed of the spatial flow is faster on the shell's side of a freely falling test mass than it is on its outer side, and this causes a temporal acceleration of the test mass towards the shell (it doesn't matter whether it's a spatial inflow or outflow).

As one enters the outer surface of the mass shell, the spatial acceleration of the flow (the difference between the flow velocities at two nearby radially separated points) is still directed towards the center of the shell (for the typical shell we are considering). However, by spherical symmetry, the flow must eventually come to rest everywhere inside the empty cavity. This means that its speed must be zero at the inner surface of the cavity at radius a and that, consequently, the flow must begin to decelerate (spatially) at some interior distance into the shell as one approaches the cavity. This spatial deceleration in the direction of the cavity (increased speed as you move away from the cavity) will, presumably, cause a temporal *deceleration* of any test mass falling towards the cavity (think of a test mass falling in an antipodal hole which has been drilled through the center of the shell). This is in stark contrast to Newtonian action-at-a-distance theory which predicts a continued temporal *acceleration* of the test mass all the way to the inner cavity surface.

We can create a model of this behavior in potential theory by identifying the square of the speed of the spatial flow $\mathbf{w}$ with a constant minus twice the potential (as we have done in previous work):

$$w^2 = \text{const.} - 2\psi \quad . \tag{4-1}$$

Our (steady state) field equation is still Poisson's equation (3-1), but now we must choose our homogeneous solution ψ_H so the physical solution ψ agrees with the boundary conditions which correspond to the spatial flow behavior as we have outlined it above.

We already know that $w^2 = 0$ everywhere inside the empty cavity region $(0 \le r < a)$ and that $w^2 = 2GM/r$ everywhere outside the shell $(r > b)$. Within the shell $(a \le r \le b)$, the homogeneous solution ψ_H must be chosen so that the square of the speed of spatial flow w^2 connects up continuously with the values already determined for the cavity region and the exterior region (in other words, we are making the fundamental assumption that the flow of space is continuous). Laplace's equation with complete spherical symmetry has the form

$$\text{lap}\,\psi_H \;=\; \frac{d^2\psi_H}{dr^2} + \frac{2}{r}\frac{d\psi_H}{dr} \;=\; 0 \quad , \tag{4-2}$$

and this has the general solution

$$\psi_H(r) \;=\; \frac{c_1}{r} \;+\; c_2 \qquad , \tag{4-3}$$

where c_1 and c_2 are constants. Given the particular integral solution (3-8b), one can proceed to determine that the constant in equation (4-1) is zero, that the appropriate homogeneous function within the shell is

$$\psi_H(a \le r \le b) \;=\; -2\pi G\mu\left(a(a+b) - \frac{ab(a+b)}{r} \right) \;, \tag{4-4}$$

and that the square of the speed of the spatial flow associated with a homogeneous hollow spherical shell is as follows.

<u>outside the shell</u> $(r > b)$:

$$w^2(r > b) \;=\; \frac{2GM}{r} \;=\; 4\pi G\mu \frac{2(b^3 - a^3)}{3r} \tag{4-5a}$$

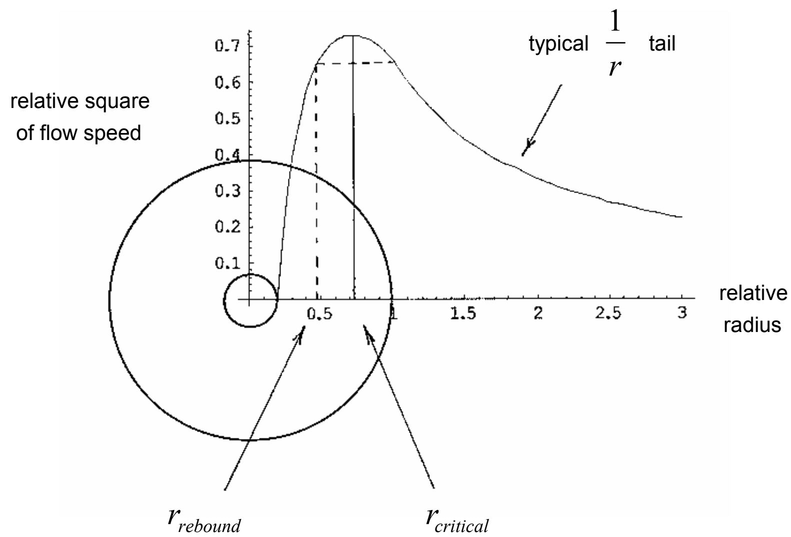
<u>within the shell</u> $(a \le r \le b)$:

$$w^2(a \le r \le b) \;=\; 4\pi G\mu\left(b^2 - \frac{2a^3}{3r} - \frac{r^2}{3} + a(a+b) - \frac{ab(a+b)}{r} \right) \tag{4-5b}$$

<u>inside the cavity of the shell</u> $(0 \le r < a)$:

$$w^2(0 \le r < a) \;=\; 0 \qquad . \tag{4-5c}$$

We can obtain a qualitative view of this spatial flow by dropping the $4\pi G\mu$ factor and plotting the relative square of its speed normalized so that the outer radius of the shell is $b \equiv 1$ and the cavity radius is again $a \equiv 0.2$:



(4-6)

One can see how the flow speed increases as one approaches the shell from the right, how it continues to increase as one enters the shell at the relative radius $r = 1$, and how it continues to do so until it reaches the critical point $r_{critical}$. At this point, the flow speed begins to decrease, and it continues to decrease all the way down to the cavity at the relative radius $r = 0.2$. At this point, the flow speed must be zero (necessitated by spherical symmetry).

Because of this flow structure, a test mass falling from rest at the outer surface into a antipodal hole which has been drilled through the center of the shell will accelerate (gravitational attraction) until it reaches the critical point $r_{critical}$. At this point, it will begin to decelerate (gravitational *repulsion*). The repulsion is so great in the region to the left of $r_{critical}$, that the test mass which has been released at the surface will not make it to the cavity at all. It will *rebound* at the point $r_{rebound}$ and return to the surface. One can see from the shape of the relative flow speed squared curve that this test mass will experience quasi-harmonic motion between its rebounding point and the surface point (the curve is, after all, proportional to the negative of the gravitational potential (see also Section 5)). This behavior is obviously entirely different than the behavior predicted by ordinary Newtonian action-at-a-distance theory. It is what is basically implied by *any* spatial flow theory of gravity, regardless of the mathematical details, just by the boundary conditions necessitated by the spherical symmetry of the flow.

The gravitational field $\mathbf{g} \equiv g(r)\hat{\mathbf{e}}_r$ caused by the flow (4-5) can be determined by means of equations (3-2) and (4-1).

outside the shell $(r > b)$:

$$g(r > b) \;=\; -\frac{GM}{r^2} \;=\; -\,2\pi\, G\mu \frac{2(b^3 - a^3)}{3\,r^2} \tag{4-7a}$$

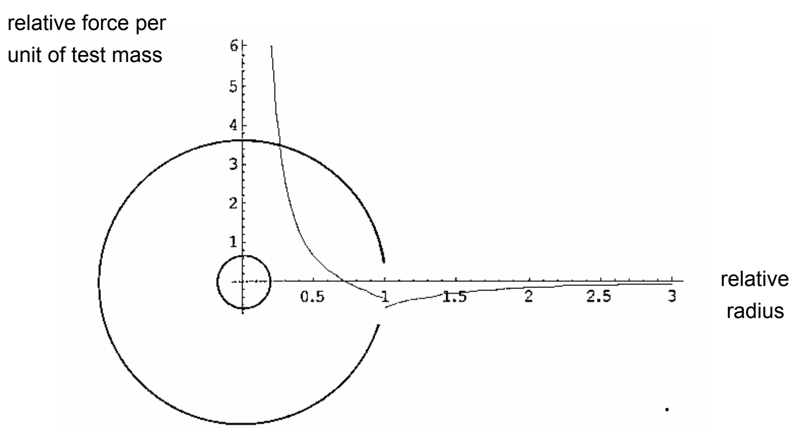
within the shell $(a \le r \le b)$:

$$g(a \le r \le b) \;=\; 2\pi\, G\mu \left(\frac{2a^3}{3r^2} \;-\; \frac{2}{3}r \;+\; \frac{ab(a+b)}{r^2} \right) \tag{4-7b}$$

inside the cavity of the shell $(0 \le r < a)$:

$$g(0 \le r < a) \;=\; 0 \qquad . \tag{4-7c}$$

We can gain further insight into this gravitational field by dropping the $2\pi\, G\mu$ factor and plotting the relative force per unit of test mass normalized so that $b \equiv 1$ and $a \equiv 0.2$:

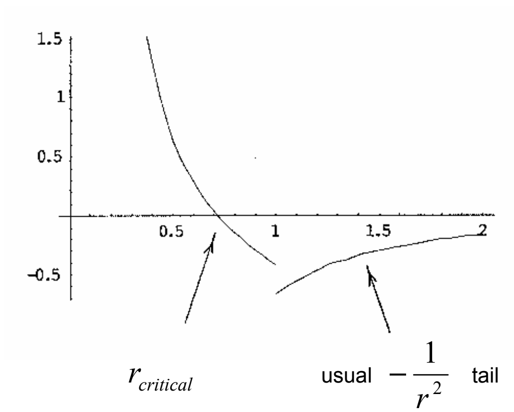


(4-8)

We observe two remarkable features. The first is that the maximum *repulsive* force, which occurs at the cavity surface, is more than six times the strength of the maximum *attractive* force which occurs just outside the outer surface (in this particular example with $a \equiv 0.2$). This startling and unexpected behavior will always be a characteristic feature in any spatial flow theory, because the drop from maximum flow speed at $r_{critical}$ to zero speed at the cavity surface will always be significantly greater than the rise in the flow speed from the outer surface to $r_{critical}$. The difference between the maximum repulsive force and the maximum attractive force is enhanced as the relative size of the cavity is diminished.

The second surprising feature in figure (4-8) is that the gravitational force field $\mathbf{g}$ is *discontinuous* at the outer surface (the graph in figure (4-6) is continuous at $r = 1$, but not perfectly *smooth* there). This discontinuity is reminiscent of the behavior of the displacement field $\mathbf{D}$ at a dielectric interface in electromagnetic theory. (One should note that the traditional gravitational theory of the hollow spherical shell is not immune from discontinuities at the outer surface, either. We can see from figure (3-10) that the gravitational *tidal* field $\mathbf{grad\, g}$ is discontinuous in the standard action-at-a-distance model.)

When we expand the ordinate of figure (4-8) near the radius of the outer surface at $r = 1$,



(4-9)

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we can observe the usual $-GM/r^2$ attractive force outside the shell, the discontinuity of the force at the surface, the diminishing of the attractive force as the critical radius $r_{critical}$ is approached, and the increasing repulsive force to the left of the critical radius as one approaches the cavity.

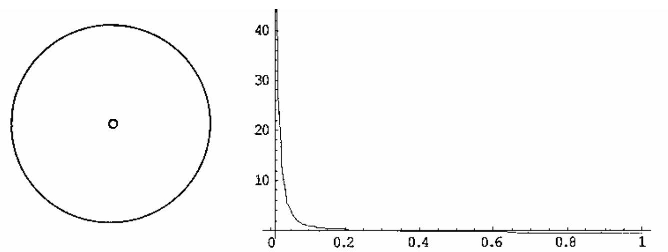
The actual size of the discontinuity in $g(r)$ at the outer surface is

$$\Delta g(r = b) = 2\pi G\mu(a + \frac{a^2}{b}) \quad , \tag{4-10}$$

and it is evident that this discontinuity can be experimentally utilized to determine the size of the cavity of the shell from measurements made at the outer surface (if the cavity is not too small). We are thinking here, of course, of the possibility of cavitated planets.

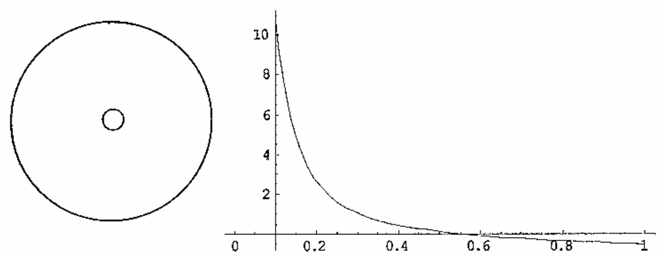
Keeping in mind that the gravitational field $g(r)$ will always be zero inside the cavity and that it will always be $-GM/r^2$ outside the shell, we plot here a little catalog of the relative gravitational fields *within* the shells whose relative cavity sizes are $a = 0.01,\ 0.1,\ 0.2,\ 0.5,$ and 0.9, successively.

$a = 0.01$



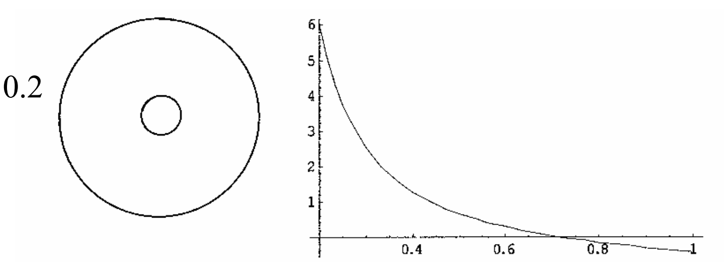
(4-11a)

$a = 0.1$



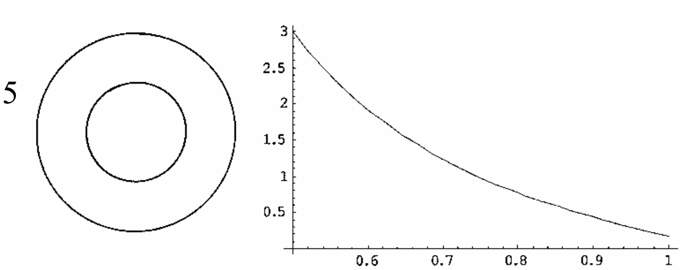
(4-11b)

$a = 0.2$



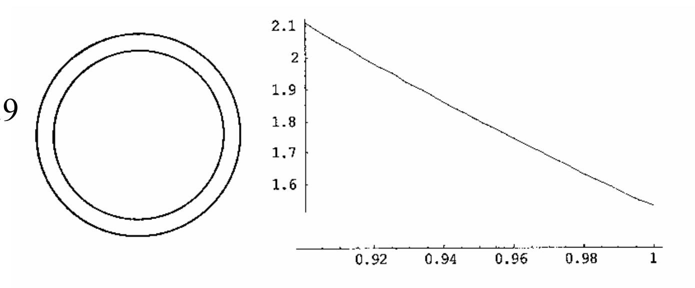
(4-11c)

$a = 0.5$



(4-11d)

$a = 0.9$

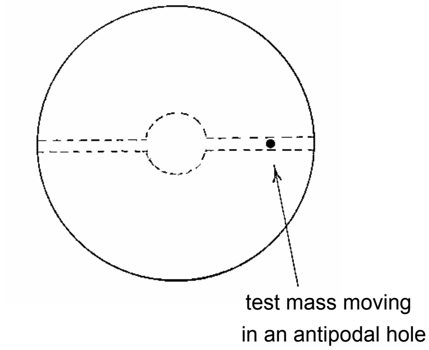


(4-11e)

We observe that the interior gravitational forces within the shell are already completely *repulsive* in shells whose relative cavity sizes are greater than one half the size of the overall shell. If this has anything to do with physical reality (as we expect it might), it is obvious that these kinds of results will require an adjustment of some of our ideas in the foundations of astrophysics. For example, if the shell in (4-11e) consists of diffuse matter instead of solid state matter, it could be rapidly expanding in all directions. It could, in fact, be developing a material shock, because the forces on the cavity side are greater than those on the outer surface (this may be happening, for example, in the remnants of certain astrophysical objects). As an even more bizarre example, a cavitated solid state planet or an astrophysical object whose dimensions are similar to those of the shell in (4-11d) might find itself exploding.

5. The Hollow Shell Cavendish Experiment

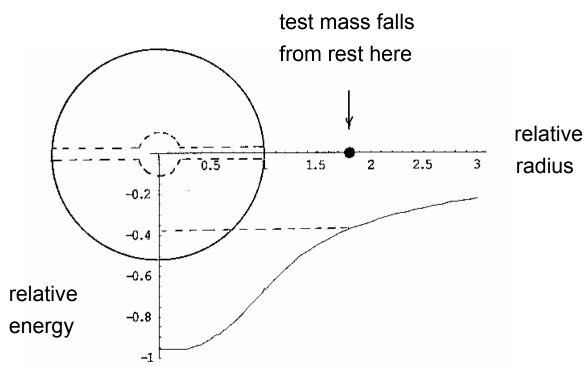
The action-at-a-distance and spatial flow theories predict identical gravitational fields *outside* spherically symmetric objects. Thus, we will be driven to explore the *interior* gravitational fields of such objects in order to empirically discern which of the two theories is closer to the truth. We are particularly interested in carrying out experiments on the interior fields of laboratory-sized objects, and for this purpose, we have in mind the probing of a hollow spherical shell of matter with a test mass which is free to move (ideally without friction) inside an antipodal hole which has been drilled through the shell's center. The hole must be bored as small as possible so that it doesn't disturb the shell's spherical symmetry too much:



(5-1)

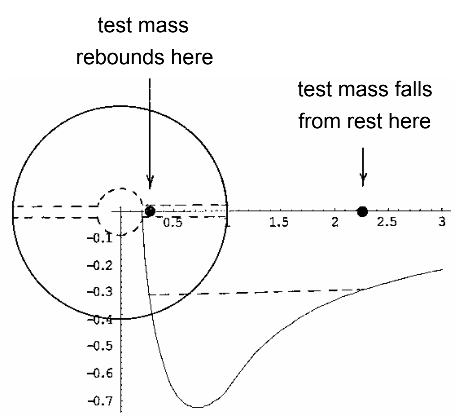
In the broadest sense, any experiment which investigates the gravitational interaction of laboratory-sized masses externally or internally is an experiment in the tradition of Cavendish [11]. One can imagine, however, an actual Cavendish balance arm being utilized to support the test mass in certain versions of this experiment [12]. Other realizations are possible as well. It is our purpose in this Section to simply point out the qualitative differences in the motion of the test mass which can be expected from the gravitational forces which arise in the two contrasting theories. We can achieve this immediately by re-examining the differences in the potential energy curves (3-9) and (4-6):

ACTION-AT-A-DISTANCE



(5-2a)

SPATIAL FLOW



(5-2b)

The straight dashed lines are the usual lines of conserved total test mass energy. The difference in the ordinates of the dashed line and the potential energy curve at any abscissa point is proportional to the kinetic energy of the test mass with respect to the shell at that abscissa point (this gives us an idea about its relative speed with respect to the shell at that point). As the dashed line is moved upwards towards the abscissa, it approaches the behavior of a test mass released from rest at infinity. When the line is above the abscissa, the test mass is not bound by the gravitational potential.

In the action-at-a-distance case (5-2a), it is obvious from the shape of the potential energy curve that the test mass will experience quasi-harmonic motion through the entire diameter of the shell between its point of release and its corresponding rebound point which is located at the diametrically opposite position on the other side of the shell. A particle released from inside the shell close to the cavity surface will oscillate very slowly through the cavity (its speed is constant while it is within the cavity). This overall action-at-a-distance behavior is very closely approximated by General Relativity for the case of the interior Schwarzschild solution [13].

In the spatial flow case (5-2b), the test mass will also experience quasi-harmonic motion between its point of release and its corresponding rebound point, but this rebound point is always located at a greater distance from the center of the shell than the cavity surface for any gravitationally bound test mass, and the resulting oscillation is *entirely restricted to one side of the shell*. The only test masses which can reach the cavity at all are those whose speeds with respect to the shell at its outer surface exceed the gravitational escape speed from that surface. The most spectacular difference between the action-at-a-distance case and the spatial flow case, however, is that a test mass released from inside the shell close to the cavity surface will be very forcibly *expelled entirely out of the shell*.

In the action-at-a-distance case, the optimum quasi-static ideally frictionless hollow shell Cavendish experiment would be carried out by releasing a test mass from rest at the outer surface and observing the resulting quasi-harmonic motion of the mass as it moves back and forth throughout the interior of the shell. In the spatial flow case, it would be to release a test mass from rest at a point inside the shell as close to the cavity surface as possible and then observe the mass being expelled from the shell well beyond its outer surface.

6. The Connection to General Relativity and Extraordinary States of Matter

We have seen in Section 2 that the interior gravitational fields of normal states of matter cannot be correctly characterized by General Relativity and at the same time by a spatial flow. In Sections 4 and 5, we have seen how startlingly different these interior gravitational fields can be even in the simpler case where we compared spatial flow predictions with action-at-a-distance predictions.

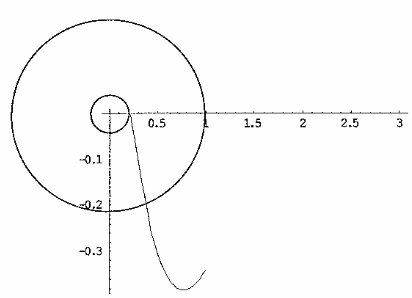
It is interesting, therefore, to note that General Relativity predicts interior gravitational fields for certain *extraordinary* states of matter which parallel the behavior of the spatial flow theory's predictions for ordinary states of matter. These General Relativistic predictions were uncovered a few decades ago in a prescient paper by Frank Tangherlini [14]. His theoretical motivations were considerably different than ours, but he nevertheless studied spherically symmetric solutions of Einstein's field equations (2-6) for the extraordinary case in which the radial stress was actually equal to the mass-energy density of the matter. One of his results was for a (non-homogeneous) hollow spherical shell, and he obtained an interior gravitational potential for it [15] which, in our notation, takes the form

$$\psi_{Tangherlini}(a \le r \le b) \;=\; -2\pi G\mu\left(\frac{a^2b^2}{r^2} - \frac{2a}{3r}(a^2 + 3b^2) + b^2 + a^2 - \frac{r^2}{3}\right)\;. \quad (6\text{-}1)$$

We can plot the relative interior Tangherlini shell potential and its relative interior gravitational field in the same normalized fashion ($a \equiv 0.2$ and $b \equiv 1$) as we have for the spatial flow case in (4-6) and (4-8) for ordinary matter:

TANGHERLINI

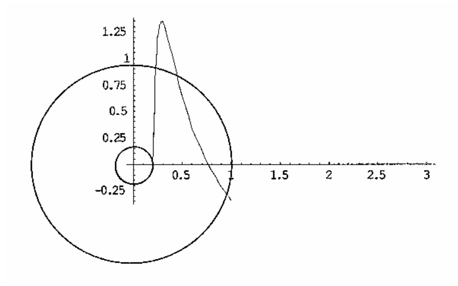
INTERIOR POTENTIAL



(6-2a)

TANGHERLINI

INTERIOR FORCE



(6-2b)

In obvious ways, they are quite similar to the spatial flow results (4-6) and (4-8). We cannot resist quoting some of Tangherlini's reactions to his hollow shell results:

> "In contrast with the potential produced by "normal" matter [in the action-at-a-distance model], the above expression for [ψ] does not simply continue to diminish as we penetrate from the outer wall of the shell to the inner wall, rather, [ψ] reaches a minimum and then increases until it attains the cavity value [$\psi = 0$]."
>
> "… that [$d\psi / dr$] vanishes at a certain critical radius r_c , within the wall of the shell, leads of course to a "repulsive" gravitational field … for the region [$a \le r \le r_c$] …"
>
> " … if a particle is released from rest outside the shell, it will not penetrate to the cavity of the shell."
>
> "On the other hand, a particle released from rest at the edge of the inner wall of the cavity … has sufficient potential energy to "fall" outward to infinity. The kinetic energy to accomplish this is of course acquired during the repulsive gravitational acceleration in the region [$a \le r \le r_c$]."
>
> " … it is also convenient to regard the region inside the shell from [$0 \le r < a$] as an inversion of the region "outside matter at infinity", *i.e.*, "the asymptotic region at infinity" is inside the shell as well as outside."

General Relativity predicts these results for extraordinary states of matter, while the spatial flow theory predicts them for ordinary matter. As we have shown in Section 2, the two possibilities are mutually exclusive for ordinary states of matter. They are not necessarily so for extraordinary states of matter.

7. Conclusion

The spatial flow predictions for the structure of the internal gravitational fields of ordinary matter may seem a little unusual. However, it is the method of physics to defer judgement until *definitive* experimental evidence is available. The hollow shell Cavendish experiment will be a relatively easy experiment to bring to fruition for this purpose. We eagerly look forward to the results. If they affirm the action-at-a-distance predictions for the internal fields, it will certainly be the swan song for any imaginable spatial flow theory of gravity. We will have to look elsewhere for Newton's gravitational "agent". On the other hand, a seemingly unlikely affirmation of the spatial flow predictions would bring an inglorious end to General Relativity as a viable theory of gravitation. Our only desire is to have the truth prevail.

Acknowledgements

Thanks to Richard Benish for turning our attention in the direction of interior gravitational fields and for providing references 13 and 14. Thanks to Henry Lindner for the insight that zero cavity flow speeds imply spatial deceleration before reaching the cavity. And thanks most of all to Harold and Helen McMaster whose generosity and moral support have made this work possible.

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